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Matrix crack evolution in multi-directional composite laminates considering thickness effects

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A probabilistic strength-based predictive model for matrix crack evolution in a multi-directional (MD) composite laminate considering thickness effect has been presented in the current work. Weibull distribution has been assumed for the *in situ* ply strength variation commonly observed in polymer composite laminates. The statistical parameters have been estimated from a master laminate. The crack density evolution has been simulated for cross-ply laminates containing varying thicknesses of 0° and 90° plies. The crack density evolution and associated stiffness degradation predictions have been compared with existing experimental values. The model has been extended to MD laminate containing plies of varying thicknesses to estimate the stiffness degradation under in-plane loading. The bounds on the stiffness have also been estimated. Good correlation is found to exist between the experimental data and simulation predictions.

Keywords: Matrix cracking; Probabilistic strength; MD laminates; Thickness effect

1. Introduction

Fiber-reinforced polymer (FRP) composites have been the primary choice for structural applications for the past four decades. These FRP's undergo multiple damage mechanisms during service. Matrix cracking is one of the prominent damage modes along with fiber breakage, longitudinal splitting, debonding and delamination [1,2]. Matrix cracking is the first damage observed and generally leads to loss of stiffness, local stress redistribution and most importantly, path for moisture or other fluid ingress leading to further reduction in composite strength or loss of its integrity [3–5]. Recently, there have been attempts to include matrix cracking and other damage-based models in the structural design [6,7].

Experimental investigation of matrix cracking and its effects on composite materials have been extensively reported and reviews on such findings are available in the literature [3,4,8]. In the context of cross-ply laminates, the ‘thickness effect’ characterizes the effect of thickness of 90° ply on the crack density evolution. Concurrently, the ‘neighboring ply effect’ characterizes the effect of varying thickness of the 0° neighboring ply on 90° ply [9–12]. The above two effects account for the ‘constraint effect’ to matrix cracking in cross-ply laminates. In a typical [0/90]_s FRP laminate, the matrix crack initiation strain decreases and evolution rate increases with increase in 90° ply thickness. In addition, the 90° ply shows more cracking in [90_m/0_n]_s laminate than [0_m/90_n]_s laminates. The reason was attributed to constraint effect of neighboring plies due to the cracking ply position wherein outer 90° ply was supported on one side by 0° ply [12]. The initiation strain for matrix cracks has

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been shown to be same in the case of $[90_m/0_n]_s$ laminate with varying 0° ply thickness. However, the crack evolution has been observed to be different, indicating the matrix crack evolution depends on neighboring ply thickness.

There have been studies to model matrix crack initiation, evolution, and its effect on stiffness. The approaches to model matrix crack initiation and evolution in composite laminates has been approached based on strength or energy [13]. Energy-based models predict initiation of a matrix crack when the total energy released by the formation of that matrix crack reaches the critical energy release rate for micro-cracking [8]. The strength-based models predict the initiation of matrix crack or any other damage in the laminate when the stress state at a material point reaches a particular strength value or satisfy some failure theory. Deterministic strength-based models usually do not agree well with experiments. However, if statistical strength distribution is considered, strength-based models are able to predict the crack evolution quite well [14]. Microscopic studies show that the matrix cracks initiate at manufacturing flaws [3,8,9,15] and the matrix crack evolution rate is associated with inhomogeneities due to statistical variation of flaws [8]. Thus, using a statistical strength distribution rather than deterministic strength enables to capture the *in situ* variation in lamina strength. Most of the studies reported have been limited to cross-ply laminates. Recently, such approaches have been extended to MD laminates [16–20]. In order to apply the above approaches, stress analysis on cracked laminate needs to be performed accurately. Various methods have been developed viz., shear lag analysis [21], variational method [22], plane strain analysis [23], 3D laminate theory [24], and finite element analysis [25].

Stress analysis coupled with Weibull statistics to account for the variation in the strength has been applied to estimate the matrix cracking evolution under static loading [14,26–30] and good agreement with the experimental data has been reported. Weibull parameters were generally estimated from curve fitting a ‘master’ or reference laminate’s experimental crack density evolution values under static loading [31–33]. Discrepancies were observed when Weibull parameters estimated from a particular thickness ‘master-ply’ were used to predict the matrix cracking in a laminate with varying traverse ply thicknesses. These models had predicted exactly same crack initiation strain for different thickness transverse 90° ply whereas decrease in cracking strain with the thickness increase have been observed experimentally [34]. In general, strength of the laminates has been estimated using UD coupon test with fibers aligned in particular orientation. However, such strength has not been able to predict the behavior of MD laminates due to constraining effect.

Various attempts have been made to account for such *in situ* variations in strength. Studies have shown that the *in situ* strength of the plies depends upon its thickness, neighboring ply thickness, and lay-up sequence of the composite [35]. This *in situ* strength decreases with the increase in ply thickness and ultimately reaches the UD strength beyond certain thickness. As high as 2–3 times increase in strength has been estimated for cross-ply laminates [36]. The *in situ* strength of T300/934 carbon fiber composites (CFC) with $[\pm\theta/90_n]_s$ sequence has been estimated using volume-dependent strength scaling law proposed by Weibull and modifying the Weibull scale parameter to account for different thicknesses [35]. There have been deviations from the experimental strength values. With the modification of Weibull shape parameter for orientation of neighboring ply along with volume-dependent scale parameter, excellent agreement has been observed with experimental strength values. Effect of volume of the stressed material on the transverse strength of AS4/3501-6 CFC material has been carried out experimentally [37] and Weibull statistics has been employed to analyze the data. From large set of experimental observations, the Weibull shape parameter has been observed to be independent of width and thickness of

the plies chosen. Good correlation with experimental data has been observed justifying the efficacy of volume-dependent scale parameter. Flaggs et al. [35] used appropriate scaling law for T300/934 laminate and concluded that appropriate scaling law may yield good prediction without shape parameter correction. Normal distribution for strength variation has also been used for matrix crack simulation [38]. Volume scaling law-based Weibull critical strain energy release rate has been used to estimate the matrix cracking in cross-ply laminate [39] and good correlation with experimental crack evolution was observed.

Numerical simulations have diverged from the experimental observations at very low and higher crack densities. The presence of defects that attributed to the low fracture stress has been postulated for the divergence from experimentally reported crack densities at low crack densities. At higher crack densities, the deviation was always higher than the experimental values reported. This is attributed to the development of delamination at the crack tip [38].

There are very few experimental observations on the statistical distribution of crack spacing or density. The crack spacing statistics has been assumed to follow exponential distribution [30] or Weibull distribution by curve fitting experimental X-ray radiographic results [40]. The stiffness reduction due to cracking has been generally estimated based on the average crack spacing from simulations. It has been shown that average crack spacing is sufficient to estimate the mechanical property reduction by comparing the experimental values with and without statistical variation of crack spacing [41]. FE-based numerical simulations have been carried out to study the effect of randomness in crack spacing on stress distribution in a cracked laminate [40]. It has been shown that the random cracking behavior can only be captured by a random crack spacing and not by an average value of crack spacing. Explicitly accounting for randomness in the crack spacing yields the lower and upper bounds on the mechanical property degradation that would be of immense use for practical design applications [40]. Consolidated experimental observation on various cross-ply composite material stiffness degradation due to matrix cracking has been reported [42]. The stiffness degradation happens at faster rate at lower crack densities and asymptotically attains a constant value with increasing load levels. Using various stiffness degradation models, the reduced properties are not “that” sensitive to higher crack densities with stiffness values approaching the ply discount value, which assumes the cracked layer contribution to transverse and shear properties is almost zero [43].

In the current work, an attempt has been made to use Weibull volume scaling law to account for the thickness effect on matrix cracking evolution in MD laminates. Oblique coordinate-based shear lag analysis [44,45] has been employed for the stress analysis of the cracked laminate. The crack evolution has been simulated using *in situ* probabilistic strength-based failure criterion. The observed crack spacing has been fitted using Weibull statistics. The corresponding reduction in stiffness values have been estimated at the average, 25 and 75% probability levels on quantile function using available models [46]. The simulations have been carried out on different laminates for which matrix cracking and associated stiffness degradation experimental data was available. The simulation has been carried out on a cross-ply laminate to verify its applicability to the change in ply thickness and constraint ply thickness. The same approach has been extended to MD laminates with different off-axis fiber orientations.

2. Matrix cracking evolution model

This section describes the key concepts and methodology used to predict matrix crack evolution in a symmetric MD laminate under in-plane loading.

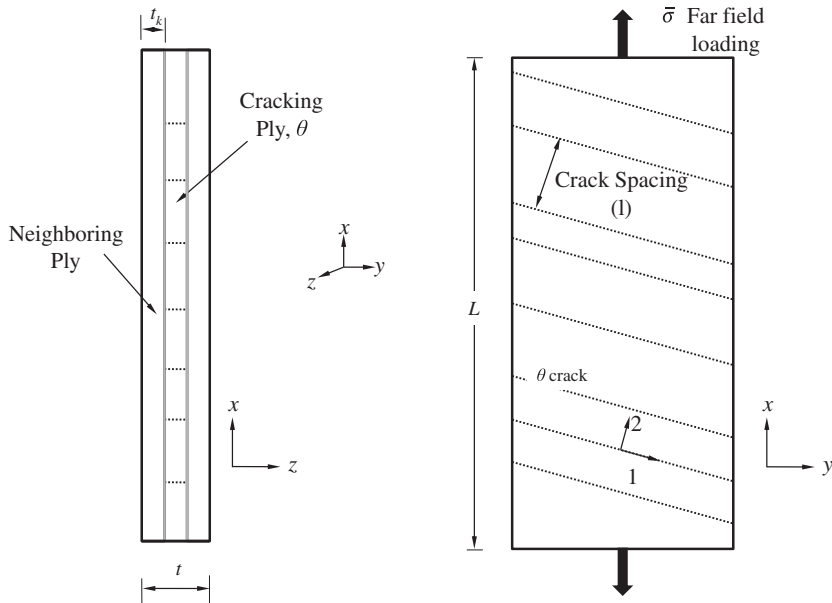


Figure 1. Matrix cracking in a MD laminate.

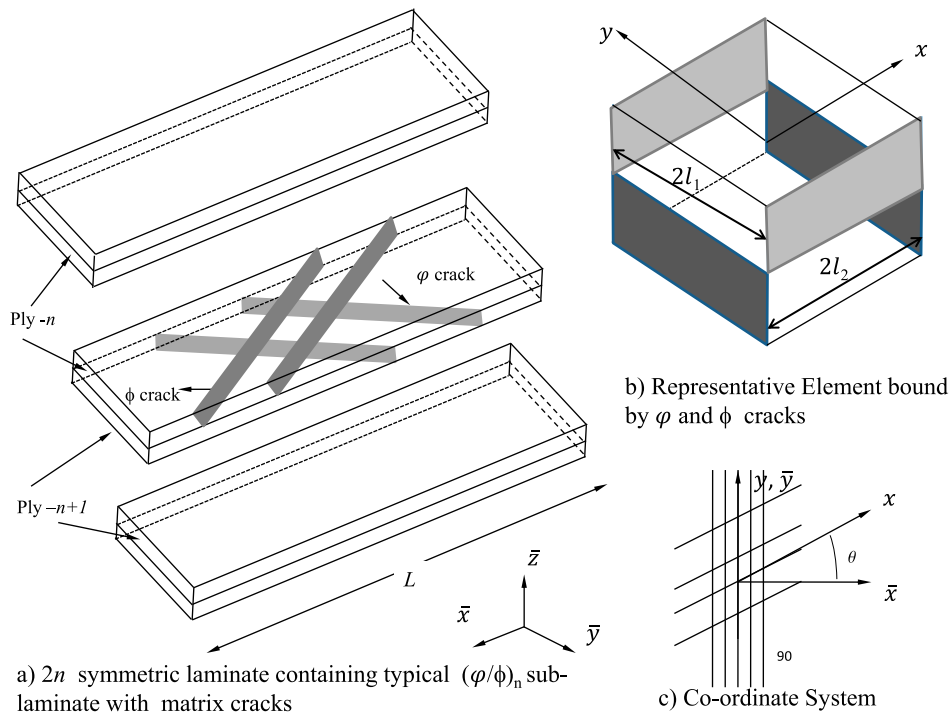


Figure 2. Representation of an cracked MD laminate.

Table 1. Material properties for laminates used in the analysis.

Property	E-Glass/epoxy [47]	Glass/epoxy [34]
Ply Thickness, mm	0.127	0.14
$E_{11}(GPa)$	42.5	44.73
$E_{22}(GPa)$	13.3	12.76
$G_{12}(GPa)$	5.8	5.8
$G_{23}(GPa)$	4.68	4.49
ν_{12}	0.3	0.297
ν_{23}	0.42	0.42
Length	100	100

2.1. Stress analysis of cracked laminate

An oblique coordinate-based stress analysis procedure described in [44] has been used to estimate the stresses in the cracked layer. Cracks in a typical ply forms along the fiber direction and transverse distance between two parallel cracks is termed as crack spacing (reciprocal of the same is crack density) (cf. Figure 1). The matrix cracking in a MD laminate is complex and random, and at high crack densities, interaction effects occur [3,45]. Other type of damage modes such as delamination has also been observed at the higher crack densities. Generally, matrix cracks form instantaneously and stress concentration occurs at its tips. Handling such effects in an analytical framework is fairly complex. In order to keep the formulation simple, such effects have been not considered in the present model.

A $2n$ ply symmetric laminate with any arbitrary orientation and different thickness in each ply has been chosen in the present study. In order to estimate stresses in the multi-layer cracked MD laminate, a modified procedure has been developed earlier and the detailed description of the same can be found in [20]. The cracked MD laminate has been divided and grouped into sublaminates for analytically estimating the stress distribution. Each sublaminate consists of any arbitrary $[\phi_m/\varphi_n]_s$ type orientation as shown in Figure 2. The representative element (RE) has been chosen as the volume enclosed by the neighboring cracks in the adjacent two plies of the sublaminate (cf. Figure 2(b)). At first, the sublaminate has been transformed into $[\theta_m/90_n]_s$ type laminate and the steps involved in the stress analysis of cracked laminate have been described in [44]. The coordinate used for the following analysis is shown in Figure 2(c). Orthogonal coordinate system has been represented by $\bar{x} - \bar{y} - \bar{z}$ and $x - y - z$ represents the oblique coordinate system. The equilibrium equations for (n^{th}) and $(n+1)^{th}$ ply (represented by 1 and 2) of a sublaminate can be expressed using shear-lag analysis. The relation between in-plane displacements in each ply is related to out-of-plane shear stresses at the interface using shear-lag analysis. The displacement fields were determined using standard elasticity approach using the eigen value formulation. The strain and stress distribution in the cracked RE has been estimated from the full field displacement fields.

2.2. Material properties

$[0/90_2]_s$, $[0/90_4]_s$, and $[0_2/90_2]_s$ lay-ups with E-glass/epoxy [47] and $[\pm\theta/90_4]_s$ plies with glass/epoxy [34] have been used in the present work. The material properties used in the simulations are shown in Table 1.

2.3. Strength distribution and volume effects

As discussed earlier, many researchers have observed the statistical variation of transverse strength of a ply. Weibull distribution has been successfully used to describe such variations in strength. The probability of failure of a typical ply in the laminate can be expressed as follows:

$$P(Y_t) = 1 - \exp \left[- \left(\frac{Y_t}{\beta} \right)^m \right] \quad (1)$$

where, β is the scale parameter, m is the shape parameter, and Y_t is the transverse strength of the ply.

As discussed in the previous section, the volume-dependent variation of strength has also been observed. The UD transverse strength estimated from the uniaxial tests may not represent the true strength of the material. In order to estimate the *in situ* Weibull strength parameter, an inverse estimation procedure has been adopted. The Weibull shape and scale parameters are estimated by trial and error using the experimental crack evolution curves available for a particular material system, called as ‘master curve.’ These Weibull parameters are then used to predict the crack evolution in other lay-ups in that material system. For example, $[0/90]_s$ laminate has been used as a laminate to estimate the Weibull parameters for AS4/3501-6 material system [37]. The crack initiation and evolution in other lay-ups composed of the same material system have been simulated using the Weibull parameters estimated from the $[0/90]_s$ ‘master curve’ laminate. In order to account for volume effects, the Weibull parameter estimated for one particular thickness has to be suitably modified to handle the thickness change in the cracking ply under consideration and the thickness of the neighboring ply. The following key concepts are utilized based on the experimental observation from the literature and as discussed in the introduction.

- (1) The Weibull shape parameter m has been assumed as a material parameter and independent of any geometrical arrangement of the laminate.
- (2) The increase in thickness of the 90° ply in a cross-ply laminate decreases the crack initiation strain while the neighboring 0° ply thickness is constant. However, the crack initiation strains are unaltered if neighboring 0° ply thickness changes while 90° ply thickness constant. From the above observations, the *in situ* strength for crack initiation of the ply strongly depends on the thickness of the ply under consideration for cracking and does not depend on the constraint ply. Hence, Weibull scale parameter for a particular ply has to be modified accounting for the thickness effect of that ply alone; the neighboring ply thickness has been ignored.
- (3) The crack evolution rate has been modified due to the constraint effect (either due to change in thickness of cracking ply or neighboring ply). It is assumed that the evolution rate is purely a function of the stress distribution inside each ply and it depends on the elastic properties, thickness, and the local stresses due to matrix crack.

In order to account for thickness effects for a particular ply, the Weibull scale parameter has been modified for volume effects. Based on the original concept coined by Weibull [48], matrix cracking in different thickness laminae has been evaluated.

$$\left(\frac{\beta_{ref}}{\beta_t} \right) = \left(\frac{V_t}{V_{ref}} \right)^{\frac{1}{m}} \quad (2)$$

Here, β_{ref} is the Weibull scale parameter estimated from master-ply for a particular volume (thickness x width x length) of the ply V_{ref} . β_t is the Weibull scale parameter of the ply for

any arbitrary volume V_t . From the experimental observations [35,37] and simulations [36], the *in situ* strength of the ply tends to asymptotically reach the UD strength of the material beyond certain thickness. However, Equation (2) estimates continuous reduction in strength with increase in ply thickness. In order take the above observations into account, the mean strength of the ply is estimated using modified Weibull parameters and if it exceeds the UD strength of the material, the Weibull parameter has been assumed same for all the thickness above such limits.

2.4. Crack spacing distribution and stiffness degradation

There have been studies to understand the random nature of matrix crack distribution. In an earlier study, the matrix cracks were assumed to follow exponential distribution [30]. However, from experimental observations, it was shown that Weibull distribution may best represent the random matrix cracking pattern [40]. The probability of crack spacing in a typical lamina may be expressed using Weibull distribution as follows:

$$P(l) = 1 - \exp \left[- \left(\frac{l}{l_0} \right)^{m_1} \right] \quad (3)$$

where, l_0 is the scale parameter, m_1 is the shape parameter, and l is the crack spacing in the ply.

Typically, the average crack spacing has been used to estimate the stiffness degradation in cracked laminates. The stiffness values are experimentally obtained by placing 50 mm or less gage length extensometer for axial strain measurement and 3 – 6 mm strain gages for transverse strain measurements [49]. In such scenarios, the average strain measured by the sensor is not fully represented by the cracking distribution and overall stiffness degradation in larger specimens. The stiffness degradation estimated from different location of the specimen may effect the property measured. Using the probability of crack spacing and statistics, the value at which the probability of the crack spacing may be less than or equal to the specified probability or probability bounds on crack spacing can be estimated using quantile functions. 25 and 75% quantile levels have been carried out in the current simulation in order to account for such variations. The stiffness estimation for the cracked laminate has been estimated following the procedure outlined in [46]. Important mathematical equations involved in the stiffness estimation of the cracked laminate has been given in appendix of [20]. The uncracked laminate properties are expressed with a subscript 0 (e.g., E_{xx0}).

3. Numerical implementation of matrix crack evolution model

Numerical implementation of the matrix cracking evolution in a symmetric MD laminate is outlined below:

- (1) A $2n$ ply symmetric MD laminate with thickness t and length L subjected to a uniform in-plane external stress field $\bar{\sigma}$ has been chosen for the present analysis (cf. Figure 1). Orthotropic material properties have been assumed for each ply. In order to start the crack evolution simulation, an initial crack spacing equal to the length of the specimen has been assumed for all the plies.
- (2) Each lamina has been divided into N material elements (each with uniform strengths) and the overall distribution of strength has been assumed to follow Weibull statistics. The Weibull parameters have been estimated using appropriate scaling law as discussed earlier. Cracking in the material element has been assumed to occur when

it meets the Hashin's strength criterion. The tensile matrix failure mode is given by:

$$\frac{\sigma_{22}^2}{Y_t^2} + \frac{\tau_{12}^2}{S^2} \geq 0 \quad (4)$$

where, σ_{22} and τ_{12} is the transverse normal stress and in-plane shear stress at a point. S is the in-plane shear strength of the material.

- (3) The cracking is always assumed to occur along the fiber direction for the lamina under consideration. In general, these cracks, known as 'tunneling cracks,' form instantly spanning the specimen width and thickness of the ply. For all the laminates considered in the present analysis, tunneling cracks have been assumed to occur. For 0° ply, no cracking is allowed.
- (4) The laminate has been loaded in stress increments of x MPa. At each increment, average global applied stresses in each sublaminate have been estimated using classical laminated plate theory (CLPT).
- (5) At each increment, the crack evolution has been estimated in each ply of the MD laminate as per the methodology outlined in [20]. The random crack spacing has been fitted with the Weibull statistics. The average, 25 and 75 % quantile levels have also been established. The stiffness degradation in the laminate has been computed using the procedure outlined as above.

4. Results and discussion

4.1. Weibull parameter estimation

The above procedure has been implemented in commercial software MATHEMATICA (version 9) in order to evaluate the matrix cracking in composite laminates. Initially, the number of material elements (N) and the stress increment for matrix crack evolution simulation has been fixed by convergence study. It was found that 100,000 material elements and 1 MPa stress increments were sufficient to yield converged solutions. The shape parameter m controls the rate of crack evolution and the scale parameter β decides the crack initiation strain. To estimate the Weibull parameters, an initial assumed β value was chosen and crack density curves were estimated numerically for various values of m . The m value corresponding to the highest correlation coefficient comparing the estimation and experimental master curve was chosen as the material parameter. Upon fixing the m value, the best fit β was estimated using trial and error, such that the simulated crack evolution at the lower crack densities coincided with experimental data. Figure 3 illustrates the determination of Weibull parameters by fitting the master curve of E-Glass/epoxy material of $[0/90_2]_s$ [50] laminates. These calibrated values for Weibull parameters have then been used for prediction of crack density evolution for other lay-ups.

4.2. Stiffness degradation in cross-ply laminate

As mentioned at the outset, *in situ* strength of the laminae in a laminate depends on its thickness, neighboring ply thickness, and the lay-up sequence. Keeping all the above parameters as a variable is very complex. Initially, in order to avoid the lay-up effect, cross-ply laminate has been utilized to establish the concept. As seen from the experiments carried out by Nairn [8,12], the neighboring ply thickness seems to have no effect on the crack initiation in 90° plies. Hence, it has been assumed the *in situ* strength of the laminae depends only on its volume. The Weibull shape parameter is assumed as material parameter

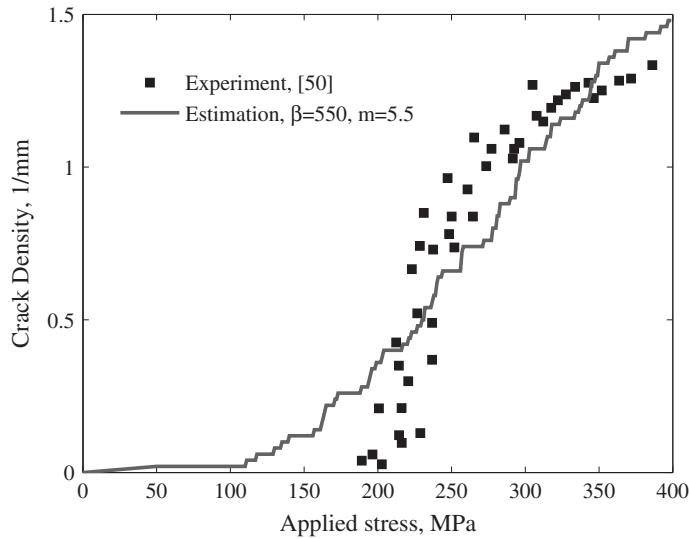


Figure 3. Estimation of Weibull parameter for $[0/90_2]_s$ [50] laminate.

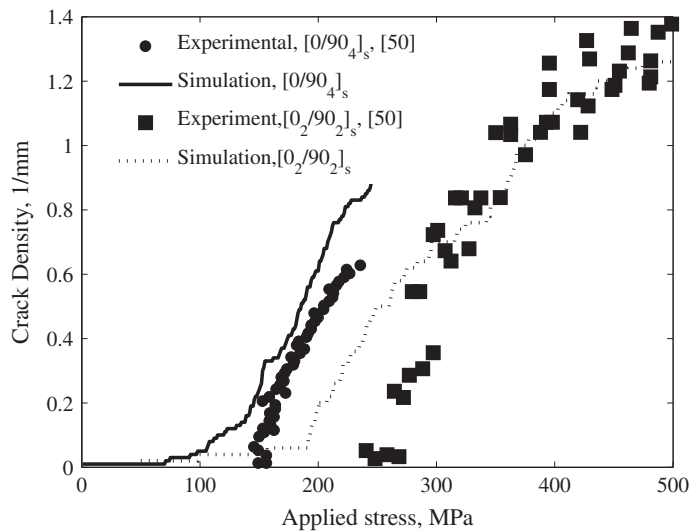


Figure 4. Crack density evolution in $[0/90_4]_s$ and $[0_2/90_2]_s$ laminate with loading.

and does not depend on any of the above variables. In order to verify the concept of thickness effect, using the master Weibull parameters, simulation has been carried out on $[0/90_4]_s$ and $[0_2/90_2]_s$ type laminate using $[0/90_2]_s$ master-ply Weibull parameters. The strength of the laminae has been modified as per Equation (2). The crack evolution simulation with loading has been shown in Figure 4. The experimental crack evolution on such material has

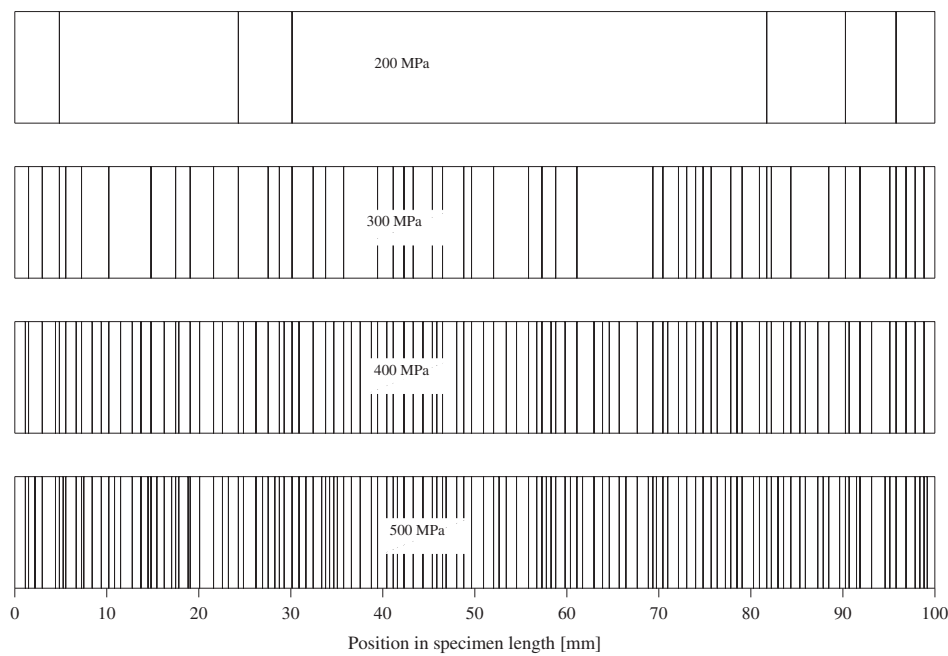


Figure 5. Crack spacing distribution along the length of $[0_2/90_2]_s$ type laminate during various stages of loading.

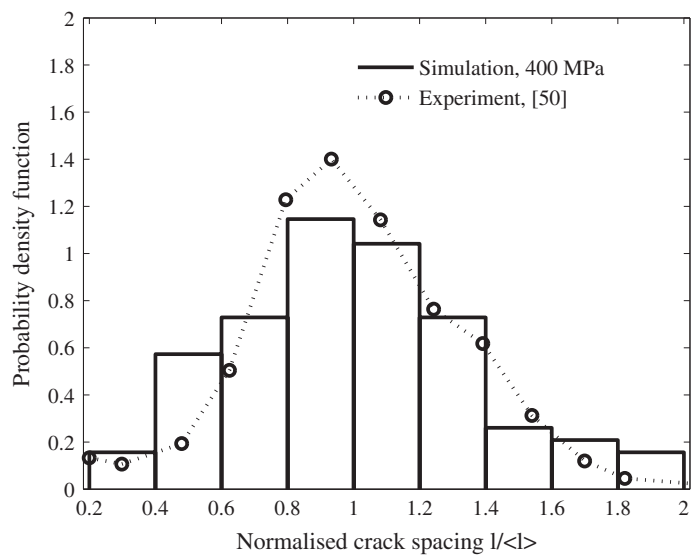


Figure 6. Comparison of experimental and simulated crack spacing in $[0_2/90_2]_s$ type laminate at 400 MPa applied stress.

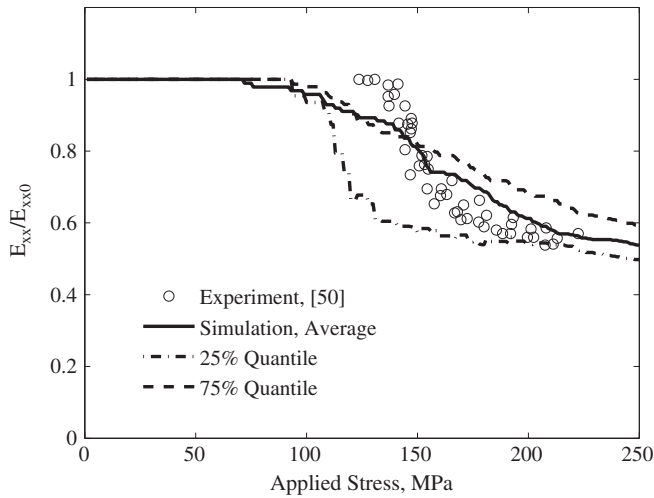


Figure 7. Normalized axial Young's modulus reduction for $[0/90_4]_s$ laminate subjected to uniaxial tensile loading.

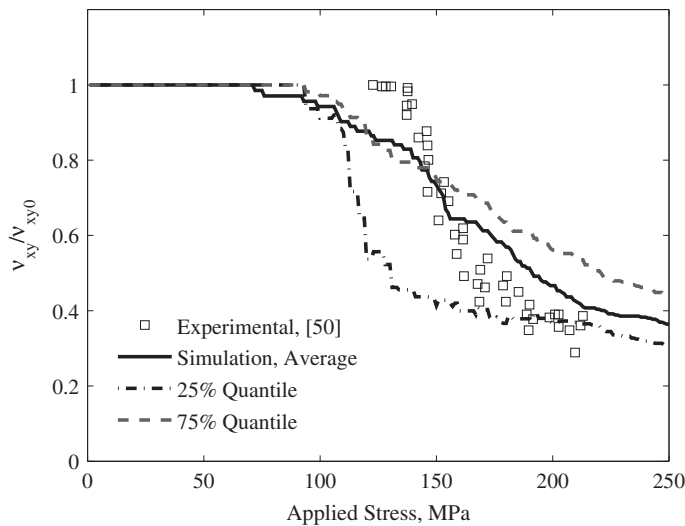


Figure 8. Normalized Poisson's ratio for $[0/90_4]_s$ laminate subjected to uniaxial tensile loading.

been carried out by Anderson et al. [50]. Close match has been observed from the current simulation with the available experimental data.

The crack evolution along the length of specimen has been estimated in a $[0_2/90_2]_s$ laminate at various load levels as shown in Figure 5. As can be inferred from the Figure 5, the cracking pattern with increasing load levels seems to be random. Consequently, the crack spacings, which determine the reduced stiffness of the laminate, are also random. It

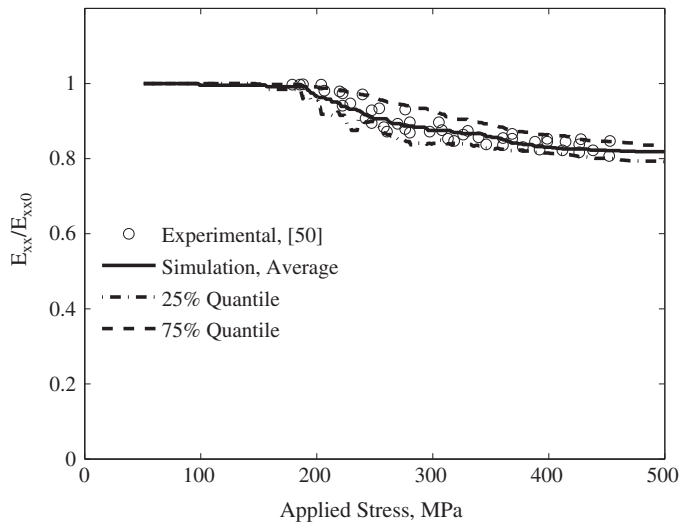


Figure 9. Normalized axial Young's modulus reduction for $[0_2/90_2]_s$ laminate subjected to uniaxial tensile loading.

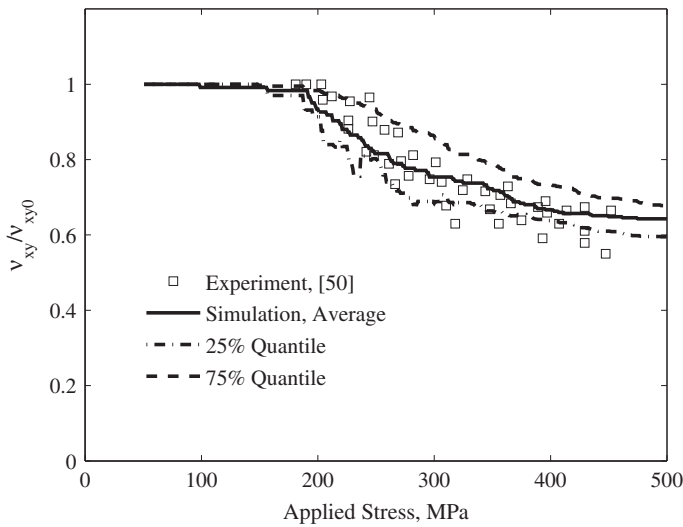


Figure 10. Normalized Poisson's ratio for $[0_2/90_2]_s$ laminate subjected to uniaxial tensile loading.

should be noted that the crack patterns are not unique to a particular laminate. Depending on the strengths assigned to each material element at the start of the analysis, different crack patterns emerge. However, similar average crack spacing and distribution has been observed from multiple simulations. The stiffness estimated from the experiment would depend on the behavior of crack spacing distribution in the measurement zone and also with specimens to specimen variability. If the stiffness is estimated in terms of bounds (corresponding to

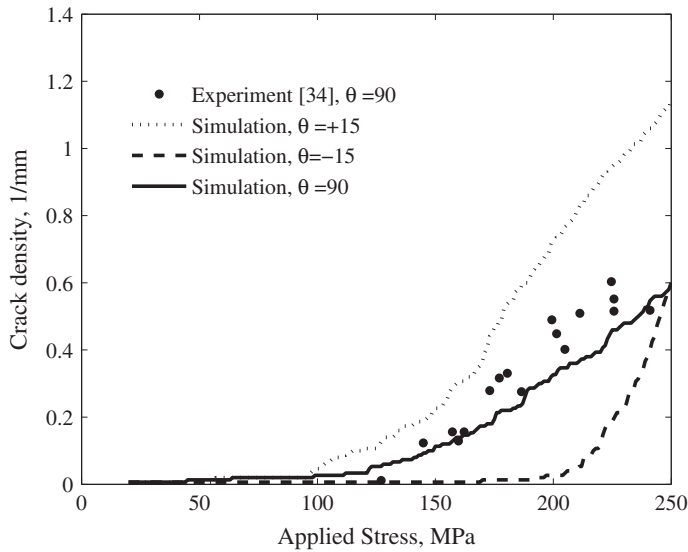


Figure 11. Crack evolution for $[\pm 15/90_4]_s$ laminate subjected to uniaxial tensile loading.

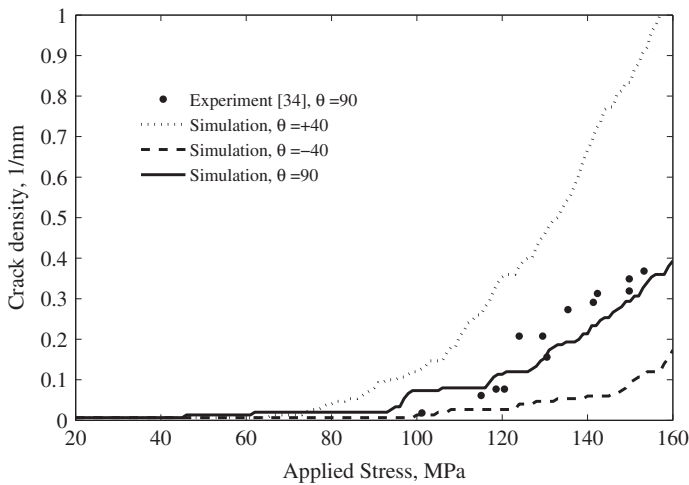


Figure 12. Crack evolution for $[\pm 40/90_4]_s$ laminate subjected to uniaxial tensile loading.

25 and 75% quantiles), some of the ambiguity associated with the lack of a single stiffness value can be addressed.

The probability density function (PDF) of crack spacing distribution at an advanced cracking stage (400 MPa) is shown in Figure 6. The experimental crack density observed has been superimposed from ref [41,47]. The current simulations show a close match with the experimental observations.

Anderson et al. [50] have reported the axial modulus and Poisson's ratio degradation with loading. The reported experimental results have been utilized to compare with current

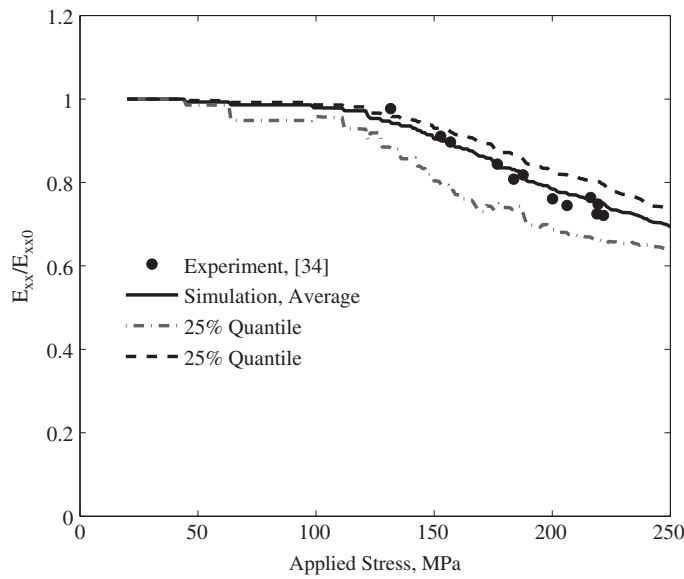


Figure 13. Normalized axial Young's modulus reduction for $[\pm 15/90_4]_s$ laminate subjected to uniaxial tensile loading.

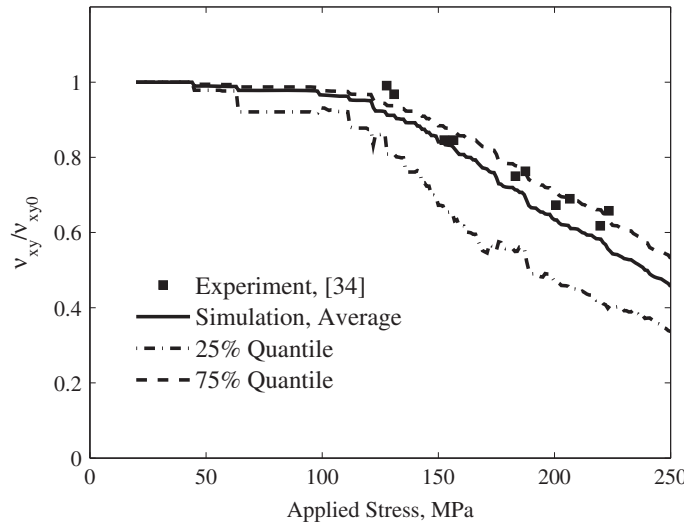


Figure 14. Normalized Poisson's ratio for $[\pm 15/90_4]_s$ laminate subjected to uniaxial tensile loading.

predictions. The axial Young's modulus and Poisson's ratio degradation estimated from the current simulation along with the experimental results is shown in Figures 7–10. The 25 and 75 % quantile level property estimate have also been indicated. The bounds are able to capture the experimental scatter in the data accurately.

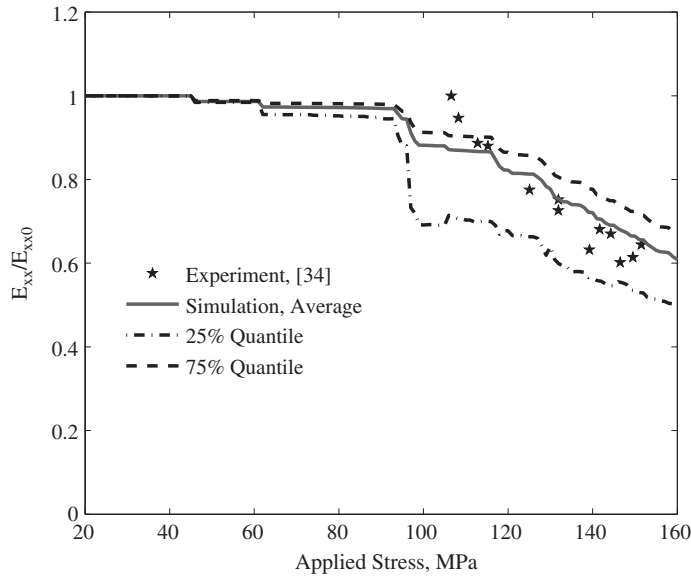


Figure 15. Normalized axial Young's modulus reduction for $[\pm 40/90_4]_s$ laminate subjected to uniaxial tensile loading.

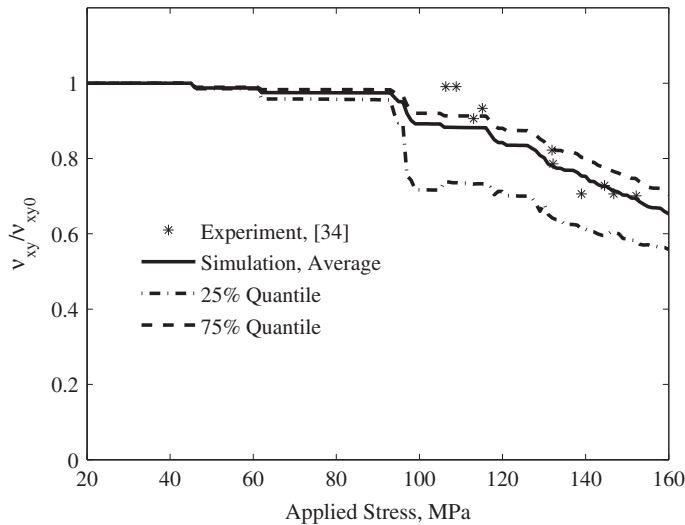


Figure 16. Normalized Poisson's ratio for $[\pm 40/90_4]_s$ laminate subjected to uniaxial tensile loading.

At higher crack densities there has been discrepancies observed, when the constraint effects are comparatively low. From experimental observation, when the constraint effect for the cracking ply is comparatively low, either in case of higher 90° plies thickness or lower neighboring ply thickness in 0° plies, delamination at the tip of 90° ply matrix crack in $[0/90]$ interface has been observed at higher cracking stages of loading [3,4,8]. These delaminations propagate on continued loading with relaxation of local stress fields and

reduces the crack evolution rate [51]. Longitudinal splitting of 0° ply due to Poisson effect at higher cracking levels have also been reported [8,11]. In the current analysis, such damage modes were not considered. However, the final goal of any such analysis is to estimate the stiffness degradation due to matrix cracking. The experimental observations and numerical simulations carried out has clearly indicate that, such deviations at higher crack densities may have negligible effect on the stiffness degradation prediction [43].

4.3. Crack evolution and stiffness reduction in MD laminates

The experimental crack evolution along with Young's modulus and Poisson's ratio degradation under axial loading for a $[\pm\theta/90_4]_s$, where $\theta = 15^\circ, 40^\circ$ type laminate was carried out by Joffe et al. [34]. Simulations on matrix cracking evolution with associated stiffness degradation on $[\pm\theta/90_4]_s$ type laminate, where $\theta = 15^\circ, 40^\circ$ have been carried out. The Weibull parameters have been estimated from identical E-Glass/ Epoxy material system with $[0/90_2]_s$ lay-up as shown in Figure 3. The matrix cracking evolution in each ply of the laminate $[+\theta]$, $[-\theta]$, and $[90]$ has been estimated following the procedure outlined in this paper. The crack evolution with loading is shown in Figures 11, 12. The overall stiffness degradation of the laminate with loading has been estimated and is shown in Figures 13–16. The degradation at 25 and 75 % quantile levels has also been simulated. Most of the experimental data has fallen within the quantile bounds. The average crack densities alone are able to predict the degradation parameters well. Good correlation exists between the experimental data and the simulation.

5. Conclusions

A volume scaling law-based Weibull strength parameter to account for thickness effect of typical cracking ply strength has been proposed. Simulation has been carried out using the proposed strength variation for crack density evolution and the associated stiffness reduction. Good correlation exists between the model and the experimental data available in the literature. Following observations can be made based on the simulation results.

- (1) The effect of ply thickness on the strength of 90° was simulated using volume scaling law. The neighboring ply thickness seemed to have negligible effect on matrix crack initiation.
- (2) The crack evolution rate variation due to neighboring ply thickness and lay-up sequence may be simulated by suitable stress analysis alone and strength parameters need not be altered.
- (3) At higher crack densities, with comparatively less constraint plies, the simulation has been off from the experimental results. However, the stiffness prediction due to such alteration may not be affected due to the limited contribution of such cracked ply on the overall stiffness of the laminate.
- (4) The volume scaling law has been able to simulate the stiffness degradation in MD laminates accurately. Hence, the proposed analytical approach can be handy for designers to conduct quick comparisons between different material lay-ups.
- (5) The crack spacing distribution has also been well captured by the current simulation and the quantile functions are able to capture the scatter in the experimental data.

Disclosure statement

No potential conflict of interest was reported by the authors.

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